REFLECTION AND REFRACTION OF ELECTRONS BY A CRYSTAL OF NICKEL

BY C. J. DAVISSON AND L. H. GERMER

BELL TELEPHONE LABORATORIES

Read before the Academy April 23, 1928

In the April number of these PROCEEDINGS[1] we reported certain preliminary results obtained in experiments in which a homogeneous beam of electrons was directed against a {111}-face of a nickel crystal at various angles of incidence, and in which observations were made on the intensity of scattering in the plane of incidence as a function of bombarding potential and direction. We had found that the incident beam of electrons is regularly but selectively reflected from the crystal face. At a given angle of incidence the reflected beam is observed whenever the speed of the incident electrons is comprised within any of certain ranges, and within each of these ranges the intensity of the beam is characterized by a sharply defined maximum. The phenomenon was interpreted as the wave mechanics analogue of the regular selective reflection of monochromatic x-rays from a crystal face.

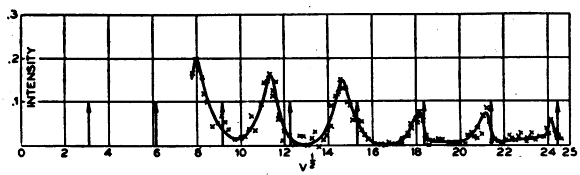


FIGURE 1

Variation of the intensity of the regularly reflected electron beam with bombarding potential, for 10° incidence—Intensity vs. $V^{1/2}$.

In the x-ray phenomenon the intensity of the reflected beam is a maximum when the wave-length of the incident beam satisfies the Bragg formula, when, that is, the wave-length has any one of the values, $\lambda = (2d/n) \cos \theta$, where d represents the distance between adjacent planes of atoms lying parallel to the surface of the crystal, θ the angle of incidence, and n any positive integer. A complete analogy between the phenomena of electron reflection and x-ray reflection would require that the Bragg formula should hold also in the case of electrons. This condition, however, is not satisfied; the wave-lengths at which the beam of reflected electrons attains its intensity maxima are not given by $\lambda = (2d/n) \cos \theta$.

This failure to conform to the Bragg law is illustrated in figure 1, which is figure 3 of our previous note (loc. cit.). Observations on the intensity of the reflected beam for angle of incidence 10 degrees are plotted in this figure against the square-root of the bombarding potential which is proportional to the speed of bombardment, and therefore to the reciprocal of the wave-length. [$\lambda = h/mv = 12.2/V^{1/2}\ A$, for electrons of moderate speed.] If the Bragg formula obtained, the maxima in this curve would occur at the values of $V^{1/2}$ given by

$$V^{1/2} = 12.2/\lambda = 12.2\left(\frac{n}{2d\cos\theta}\right)$$
$$= n \times 3.05, \text{ for } \theta = 10 \text{ deg.}, d = 2.03\ A.$$

These values are indicated by the arrows in the figure, and one notes a definite failure of the observed maxima to fall at the calculated positions.

A discrepancy of this sort was not unexpected. We had found in our first experiments[2] that electron diffraction beams do not coincide in position or in wave-length with their Laue beam analogues, and it was anticipated that the properties of the crystal responsible for these discrepancies would manifest themselves, in the case of electron reflection, in a departure from the Bragg law. On the other hand, it was expected from considerations of symmetry that the reflection would be regular, and it appears to be strictly so in all cases.

The single statement covering both reflection and diffraction is that for electrons of the speeds used in our experiments (bombarding potentials up to 600 volts) Bragg's law does not obtain; the wave-length of the beam of scattered electrons as calculated from the de Broglie formula is never the same (except in a special case to be mentioned later) as that of the corresponding beam of x-rays. It is a matter of great importance, of course, to discover the cause of this difference.

The suggestion was made by Eckart[3] on the basis of the results contained in our note to *Nature* that this failure of the Bragg law is to be ascribed to a refraction of the electron waves by the crystal. The same suggestion was made also by Bethe[4] who pointed out that such an effect could be readily accounted for on the wave theory of mechanics as a consequence of the change in potential energy experienced by the electrons on passing into the metal. This idea is a particularly attractive one as it is capable of accounting not only for wave-length and directional differences of the types observed, but is capable of accounting also for the well-established fact that the wave-length λ ($= h/mv$) of each of the diffraction beams satisfies the plane grating formula with respect to one or another of the atomic plane gratings to which the surface layer of atoms is equivalent.

Bethe calculated indices of refraction for nickel from the data contained in our note to *Nature*, and in our *Physical Review* article (loc. cit.) we made similar calculations and displayed the results in figure 18. Both of these sets of calculations were based upon the assumption that each electron beam is the analogue of the Laue beam of the same azimuth and order and of the next *longer* wave-length, this being the correlation we then favored. These calculations yielded values of refractive index less than unity. We took occasion to point out that the correlation of x-ray and electron beams which we then favored was not necessarily the correct one, that if each electron beam were assumed to be the analogue of the adjacent Laue beam of *shorter* wave-length the indices of refraction would all be greater than unity, and that such values seemed inherently more acceptable. The discrimination was based upon the consideration that if the work function of the metal is equivalent to Φ volts one might reasonably expect the index of refraction of the metal for electrons to be given by $\mu = (1 + \Phi/V)^{1/2}$, where V represents the bombarding potential. Values of μ corresponding to this alternative correlation were not, however, given in our paper, nor were values of Φ given for either correlation.

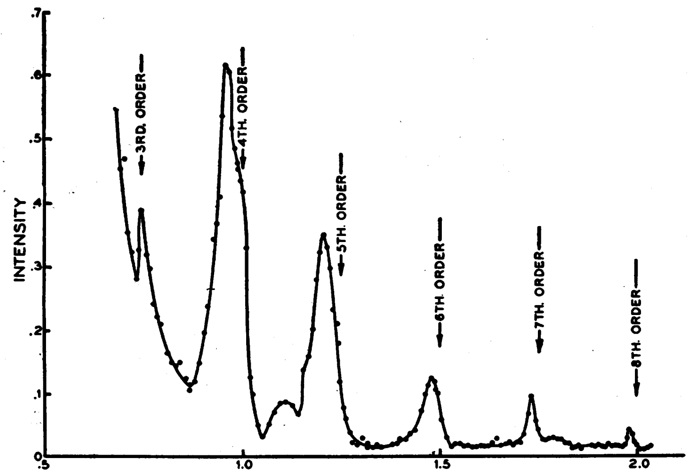
A similar expression for the refractive index, to wit, $\mu = (1-V/E)^{1/2}$, was derived by Bethe (loc. cit.) from Schroedinger's equation. In this formula E represents the total energy of the electron (bombarding potential) and V the mean potential energy of the electron inside the crystal. The zero of potential is taken outside the crystal so that a negative value is expected for V and values greater than unity for μ.

In a more recent note Bethe[5] has calculated values of V corresponding to both correlations of electron and Laue beams and finds that the values corresponding to our alternative correlation show smaller departures from their mean value than those corresponding to the correlation which we originally favored. According to Bethe's calculations the most probable value of V is -15 volts. And this value he finds acceptable on the view that it represents not the Richardson work function of the metal, but the space average of potential within the crystal, a quantity which figures in the Fermi-Sommerfeld theory of thermionic emission.

To account for the absence of the analogues of certain of the principal Laue beams in this second or alternative correlation Bethe suggests, as we also suggested in our *Physical Review* article, that these beams suffer total reflection at the surface of the crystal and therefore fail to emerge; also that the intense beam occurring in the {100}-azimuth for bombarding potential 35 volts should be regarded as a diffraction beam rather than as a "grazing beam." We differed from Bethe in thinking that these considerations were inadequate to account for all of the missing beams.

The observations on the electron reflection beams reported in this and in our previous note were undertaken to determine which, if either, of the proposed correlations is the correct one. We hoped also to determine whether or not refraction in the ordinary optical sense is actually a property of the crystal. For in spite of the theoretical considerations favoring this view, the experimental evidence supporting it was very meager. The diffraction experiments supply one value of refractive index for each of a series of electron speeds or wave-lengths, but as these, through errors of measurement or otherwise, do not form a smooth dispersion curve it was not established that the crystal is, in fact, characterized by a parameter which may properly be called its index of refraction for electrons. Our "refractive index" would lose much of the significance which has been attached to it if, for example, it were found to depend not only on wave-length but also upon angle of incidence.

The curve of figure 1 cannot be used to discriminate between the proposed correlations. If the first maximum on the left is the third of the series, the wave-length of the x-ray beam is shorter than that of its electron analogue, but if it is only the second then the wave-length is longer. The former



$\frac{1}{\lambda}$

FIGURE 2

Similar to figure 1 under better experimental conditions—Intensity of reflected beam vs. $1/\lambda$ for $\theta = 10°$.

correlation is perhaps suggested more strongly, but the situation is really the same as with the diffraction data. We do not know with certainty whether the analogue of a given electron beam is the adjacent x-ray beam of shorter wave-length or the adjacent beam of longer wave-length. The former correlation leads to values of refractive index greater than unity and the latter to values less than unity. Both sets of values are given in our previous note.

The further observations which have been made lend considerable support, we think, to the view that electron refraction in the optical sense is a property of the crystal, and that the indices are greater than unity. On the other hand the results are less simple than might be expected and more data will be required before a really definite answer can be given. For bombarding potentials above 150 volts the results so far obtained are consistent with a dispersion curve represented by $\mu = (1 + \varphi/V)^{1/2}$, the value of φ being about 18 volts. But for bombarding potentials below 150 volts and above 65 volts (the limit of our observations) the situation is much more complicated. More than one value of index is found for certain wave-lengths within this region, and it is not yet certain that this peculiarity is independent of the order of the reflection. It appears also that under certain conditions reflection occurs as if the index of refraction were unity; intensity maxima occur, the data of which satisfy the ordinary Bragg formula.

One maximum of this kind is shown very clearly in figure 2, in which we have plotted a second set of observations on the reflection beam for angle of incidence 10 degrees taken under better experimental conditions than those of figure 1. (In figure 2 the intensity has been plotted against the reciprocal of wave-length instead of against the square-root of the bombarding potential, as in figure 1.) The secondary maximum which coincides in position with the third order Bragg beam is indicated also in figure 1, although at the time these earlier observations were made we did not believe in the reality of this feature. There is an indication in figure 2 of a fourth order Bragg beam, and of other minor features, the most prominent of these being the small maximum at $1/\lambda = 1.11$ A^{-1}.

We would like to suppose that the beams which satisfy the Bragg law are due to constructive interference among the waves proceeding from the numerous crystal facets which make up the surface of the target. These facets differ in level by integral multiples of the Bragg constant d, and together constitute what amounts to an echelon grating. That the waves proceed from small lattices associated with the facets rather than from the surface layers of atoms only would seem not to invalidate this explanation.

Observations similar to those of figure 2, but less complete, have been made at other angles of incidence up to $\theta = 52$ degrees. The results of all such measurements are exhibited in figure 3. The dots in this diagram coördinate the electron wave-lengths and the cosines of the angles of incidence at which intensity maxima of the reflected beam have been observed, the area of each dot being roughly proportional to the strength of the maximum. The data corresponding to the maxima of figures 1 and 2 occur at the extreme right (abscissa 0.985). The straight lines are the graph of the Bragg formula $\lambda = (2d/n) \cos \theta$ in its various orders. If the intensity maxima of the electron reflection beam were determined in the same way as are the intensity maxima of x-ray reflection the dots would, of course, fall along these straight lines.

The observation is that the dots fall on lines and curves which display the same general characteristics as the Bragg lines; they have the same trend and are similarly spaced. The suggestion from the figure is that the dots lying just above the fourth order Bragg x-ray line determine the fourth order electron line, and that the dots which fall between the second and third order Bragg lines correspond to third order reflections. On the other hand, it is conceivable that the orders in these cases are actually the third and second rather than the fourth and third. To discriminate between these possibilities and at the same time to test the hypothesis that electron waves experience ordinary optical refraction, we have calculated and plotted four sets of quantities: The indices of refraction corresponding

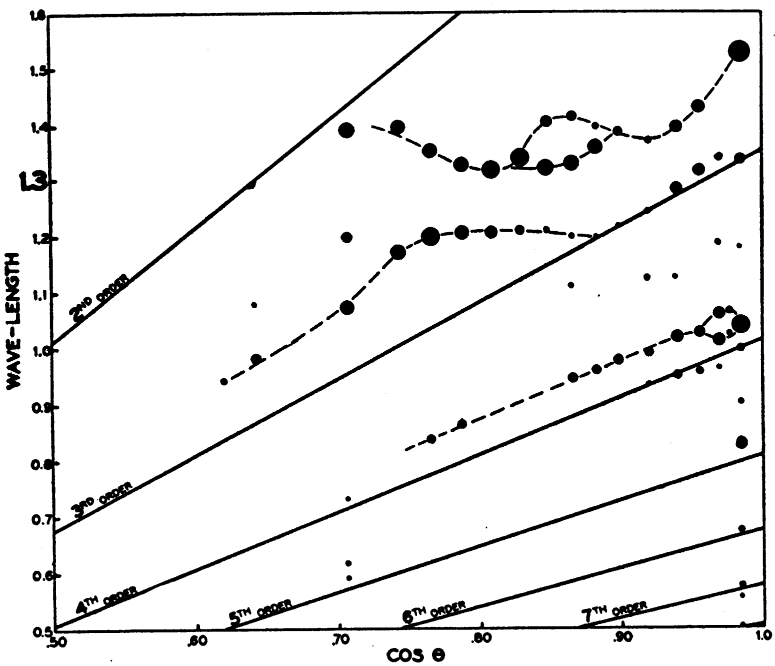


FIGURE 3

The positions of the dots on this figure correlate the wave-lengths and the angles of incidence at which electron reflection occurs from a {111}-face of a nickel crystal. The area of each dot represents the intensity of the reflection.

to the dots in figure 3 assuming the first assignment of orders mentioned above, and then the second, and also the "spacing factors" defined in our *Physical Review* article for the same two assignments. It was anticipated that one at most of these sets of quantities when plotted against wave-length would form a continuous curve. In three of the cases the curves seem to be definitely discontinuous; the curve determined by observations in one order does not join up with the curve of the next order. The remaining case is that of the refractive indices corresponding to the first assignment, and here continuity if not thoroughly established is at least strongly indicated. The formulas used in calculating indices and "spacing factors" are the following:

$$\mu^2 = n^2\lambda^2/4d^2 + \sin^2\theta, \ \beta = n\lambda/2d \cos\theta$$

The indices corresponding to the first assignment are plotted in figure 4, a different kind of symbol being used for each order of reflection. It will be noted that at $1/\lambda = 0.93\ A^{-1}$ observations in adjacent orders yield nearly the same value of index, and that this is true also at $1/\lambda = 1.02\ A^{-1}$, at $1/\lambda = 1.06\ A^{-1}$ and at $1/\lambda = 1.20\ A^{-1}$. The solid curve to the right of $1/\lambda = 1.00\ A^{-1}$ is a graph of $\mu = (1 + \varphi/V)^{1/2} = (1 + \varphi\lambda^2/150)^{1/2}$ with the value of φ adjusted to obtain as good a fit as possible with the observed points in this range. The value of φ is 18 volts and the agreement is as good, we think, as can be expected. The graph of this relation is continued to the left by the interrupted curve.

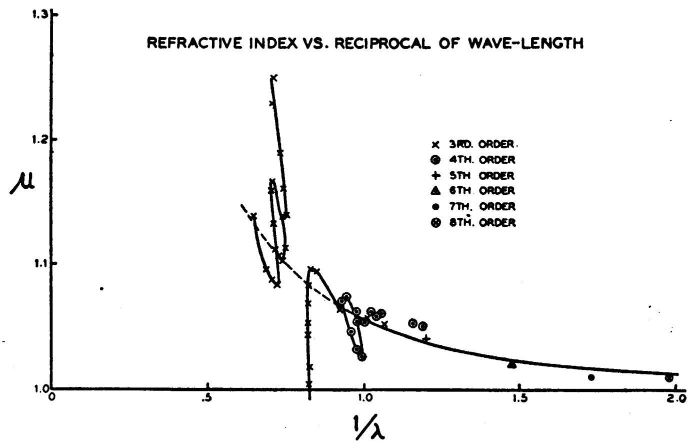


FIGURE 4

μ vs. $1/\lambda$—Refractive indices for nickel calculated from the data of figure 3.

What interpretation is to be placed upon the peculiar behavior of the index in the region below $1/\lambda = 1.00\ A^{-1}$ we do not know. The form of the curve in the region about $1/\lambda = 0.8\ A^{-1}$ suggests some sort of resonance phenomenon such as that to which we ascribe the anomalous dispersion of light. But oscillators and resonance would appear to have no place in the theory of electron refraction. Refraction in the two cases arises from quite different causes, as is evident from the fact that the trend of what is presumably the normal dispersion curve of nickel for electrons is opposite to that of the normal dispersion curves of materials for light and x-rays; in the former case $d\mu/d\lambda$ is positive while in the latter it is always negative.

We have not succeeded in identifying the position of the region of "anomalous dispersion" definitely with any of the known critical constants of nickel. The critical potentials of soft x-ray emission from nickel determined by various experimenters[6] show nothing striking near 90 volts, nor do Pétry's values of the critical potentials of secondary electron emission from nickel. The wave-length of the K x-ray absorption limit for nickel, 1.49 A, is fairly near the long wave-length edge of the "anomalous dispersion" region at 1.4 A.

In the data which we have at hand there is an indication that the bifurcation of the curves shown in figure 3, at $\lambda = 1.32\ A$, $\cos\theta = 0.082$, in the third order and at $\lambda = 1.02\ A$, $\cos\theta = 0.96$ in the fourth order, occurs again at $\lambda = 0.82\ A$, $\cos\theta = 0.98$ in the fifth order.

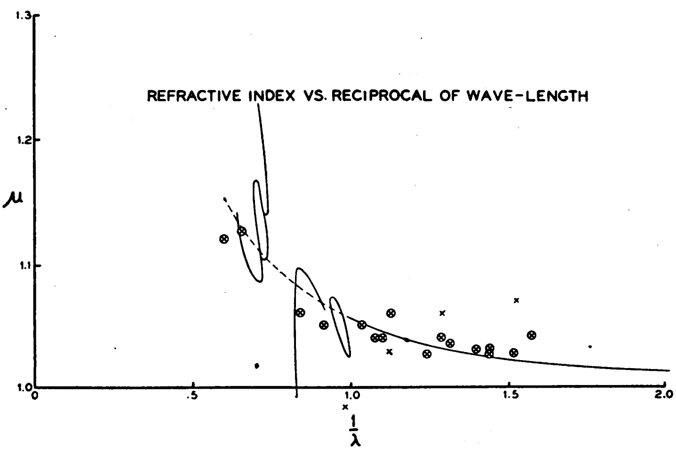


FIGURE 5

Refractive indices calculated from the data of the diffraction experiment (loc. cit.). More reliable points indicated by crossed circles, less reliable points indicated by crosses.

It is possible that the behavior of the reflection beam is not independent of the azimuth of the plane of incidence. The results given here are for the incident beam lying in one of the {111}-azimuths of the crystal as described in our previous note. We hope to look for a dependence upon azimuth at some future time.

In figure 5 we reproduce the dispersion curve of figure 4 and in the same diagram plot values of the refractive index calculated from our earlier observations on the electron diffraction beams. These calculations are, of course, based on the second correlation of electron and Laue beams. The values of μ differ somewhat from those given by Bethe because, in reducing the data, we have in each case applied a small correction to the observed angle of the beam to make wave-length and angle satisfy precisely the plane grating formula. It will be noted that, with the exception of two, all of the points fall to the right of the regions of "anomalous dispersion," and that none of them falls in this region. It is due to this circumstance presumably that the displacements of the electron diffraction beams from their x-ray analogues display no marked abnormalities. It will be noted also that although the values of μ calculated from the diffraction beams are rather scattered they are not inconsistent with the dispersion curve constructed from the more precise data of the reflection beams.

[1] Davisson and Germer, *Proc. Nat. Acad. Sci.*, **14**, 317 (1928).

[2] Davisson and Germer, *Nature*, **119**, 558 (1927); *Phys. Rev.*, **30**, 705 (1927).

[3] Eckart, *Proc. Nat. Acad. Sci.*, **13**, 460 (1927).

[4] Bethe, *Naturwiss.*, **15**, 787 (1927).

[5] Bethe, *Ibid.*, **16**, 333 (1928).

[6] Andrewes, Davies and Horton, *Proc. Roy. Soc.*, **117**, 660 (1928).

OSCILLATIONS IN IONIZED GASES

BY IRVING LANGMUIR

RESEARCH LABORATORY, GENERAL ELECTRIC CO., SCHENECTADY, N. Y.

Communicated June 21, 1928

In strongly ionized gases at low pressures, for example in the mercury arc, the free electrons have a Maxwellian velocity distribution corresponding to temperatures that may range from 5000° to 60,000°, although the mean free path of the electrons may be so great that ordinary collisions cannot bring about such a velocity distribution. Electrons accelerated from a hot cathode (primary electrons), which originally form a beam of cathode rays with uniform translational motion, rapidly acquire a random or temperature motion which must result from impulses delivered to the electrons in random directions.

In this laboratory we have been studying these phenomena[1] in detail during the last 4–5 years, but the mechanism underlying the Maxwellian distribution and its extremely short time of relaxation have not been understood. At an early date it occurred to me that electric oscillations of very high frequency and of short wave-length in the space within the tube might produce a scattering of the kind observed, but calculation showed that average field strengths of several hundred volts per centimeter would be necessary and this seemed an unreasonable assumption. Experiments capable of detecting oscillations of the electrodes with amplitudes greater than 0.2 volt failed to show such oscillations.

Ditmer[2] although unable to detect oscillations, concluded that oscilla-